Teacher's manual for First year algebra scales

Hotz Henry Gustave
Teacher's Manual
For
First Year Algebra Scales
By

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Published by
Teachers College, Columbia University
New York City
1922
PREFACE

This manual is compiled for the purpose of assisting teachers of mathematics in the administration and practical use of my First Year Algebra Scales. There is a feeling that the original monograph, which appeared in the Teachers College Contributions to Education series, is too technical and, consequently, too difficult for most teachers to read intelligently and to determine from it with ease how to apply the scales most profitably. Suggestions concerning the purpose of these scales are incorporated in this manual in as simple and direct form as possible. Special training in statistical methods is not necessary for their comprehension.

Besides the Tentative Standards of Achievement proposed in the original monograph, scores more recently obtained in various cities and through school surveys have been included. Suggestions on presentation and diagnosis of results have also been added. In order that the progress of a class may be more scientifically determined, a revision of the tests, with the exercises arranged in duplicate or alternate scales of equal difficulty, will be published in the near future.

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Teacher's Manual For
First Year Algebra Scales

The First Year Algebra Scales were first published in 1918. Since then they have been extensively used by teachers, school administrators, directors of educational research, and in various school surveys.

I. DESCRIPTION OF SCALES

The scales consist of five different sheets of algebraic exercises designed to measure the ability of pupils in elementary algebra. They are:

1. Addition and Subtraction
2. Multiplication and Division
3. Equation and Formula
4. Graphs
5. Problems

The first two scales, it will be seen, are designed to test the achievement of students in the fundamental operations, involving integral, fractional, and radical expressions; the second two, to test the ability of students in handling the instruments of quantitative thinking; while the last is composed of verbal problems of the type usually stressed in the first year of algebra.

The exercises in each scale are arranged in order of difficulty; that is, each scale begins with exercises so easy that they can be solved by practically every member of a class. Each succeeding exercise, however, becomes increasingly more difficult so that the last ones in each scale can be solved by only a relatively small number of students who try them.

Two series of each scale are offered—Series A and Series B. Series B is the longer and contains from eleven to twenty-five exercises in each scale. Series A is only about one half as long and contains from eight to twelve exercises in each scale. It covers just as wide a range of difficulty and has the added advantage of having the intervals between successive exercises and problems approximately equal: that is, Ex. 3 of a given scale is as much more difficult than Ex. 2, as Ex. 2 is more difficult than Ex. 1.

1 Hotz, Henry G.: First Year Algebra Scales. Teachers College, Columbia University, Contributions to Education, No. 90.
In determining individual and class scores, the factor of primary importance is not so much how many exercises an individual can solve correctly in a given time, but rather how far along on the scale of exercises, arranged in order of increasing difficulty, he can perform satisfactorily. In other words, the pupil is measured almost entirely by the point which he reaches on the scale. For this reason the tests may very properly be technically called "scales," and are characterized as "difficulty tests" or "power tests" by specialists in educational measurements.

The scales were derived from data obtained from tests given to over 16,000 high-school students. The schools which cooperated in standardizing the scales varied all the way from the small rural high school to the large cosmopolitan high school. Classes were tested in eighty-four high schools located in the states of Massachusetts, Connecticut, Rhode Island, New York, New Jersey, Ohio, Wisconsin, Missouri, Oklahoma, Colorado, and Washington, and the results subjected to intricate statistical treatment. The difficulty of each exercise, or its position on the scale, was determined by the percentage of pupils solving each exercise correctly.\(^2\)

2. SELECTION OF SCALES TO BE USED

Series A will be found on the whole to be more satisfactory than Series B, especially where the time available for testing purposes is limited. This is particularly true if the purpose of the test is primarily to determine degrees of attainment. If, however, the purpose of the test is mainly diagnostic, that is, to discover difficulties which the students are encountering, Series B should be used. It contains a richer variety of exercises and, consequently, a greater number of type processes. This makes it possible for teachers to make a more complete analysis of the mistakes made by pupils.

If only one scale can be used, it should be the Equation and Formula Scale, because it is more comprehensive and so tests a much wider range of functions. At least two scales should be used, however, and the scale which undoubtedly comes second in importance is the Problem Scale. If Series A is used there will be ample time to give both during a single class period of forty minutes.

\(^2\)For a complete account of the method employed in locating each exercise on a linear scale, consult Hotz, Henry G.: First Year Algebra Scales.
Whenever it is possible to do so, all five of the tests of a given series should be used, since the achievement on all of the tests gives a much more reliable indication of a pupil’s ability than the results from one or two tests would give.

Teachers have found it most practicable to use the tests in rotation somewhat as follows:

1. At the end of three months
   Addition and Subtraction Scale
   Equation and Formula Scale
2. At the end of six months
   Multiplication and Division Scale
   Problem Scale
3. At the end of nine months
   Equation and Formula Scale (repeated)
   Graph Scale

Whenever it is desired to use the same scale a second time, it is advisable to select it from a different series. It is feared that unless at least six months’ time has elapsed since a given test was used some of the practice effect may survive.

3. WHEN TO GIVE THE TESTS

The scales may be used very profitably as early as the end of the third month of the school year.

Tentative Standards of Achievement, based upon the 16,000 papers of the original study, were compiled for three-, six-, and nine-month intervals. It is, therefore, much more satisfactory for comparative purposes to submit these scales to algebra classes immediately after they have studied algebra for three, for six, or for nine months. However, data on the achievement at other intervals are being collected constantly, much of which is included in this pamphlet, and as time passes more and more information with regard to the achievement that may reasonably be expected at other intervals will become available.

The scales are not intended to be used beyond the first year. For this reason very few results from classes having had algebra more than ten months have been reported.


See pp. 27-34.
4. DIRECTIONS FOR ADMINISTERING THE TESTS

1. Preliminary. Before passing the papers see that the desks are cleared and pupils are provided with pencils. For the graph test rulers should also be provided. Then make the following statement:

"I am going to give you a test to see how well you can solve exercises in algebra. Papers will be passed to you with the printed side down. Please leave them so until I tell you to turn them over."

2. Pass the papers, or have them distributed by the pupils in the front seats, with the printed side down (Series B, first page down) and the top end away from the pupil.

3. When all are ready say to the class:

"Turn your papers. Write your name in the first blank space," etc. (The number of blank spaces to be filled out may be determined by the one giving the test. It is not necessary to have all filled in. Some teachers simply have pupils write their names in the upper right hand corner of the blank page so as to prevent the pupils from seeing any of the exercises before all of the directions have been given.)

4. Then repeat one of the following series of directions, depending upon the tests to be given:

Addition and Subtraction Scale

"Attention! The exercises on these sheets are in addition and subtraction—collection of terms. Take the exercises in the order in which they are given. Work as many as you can and be sure you get them right. Work directly on these sheets and do not ask anybody any questions. When you have worked all the exercises you can, lay aside your pencils and remain quiet so as not to disturb those who are still working. You will have twenty minutes in which to work (Series B, forty minutes). Start."

Multiplication and Division Scale

"Attention! The exercises on these sheets are in multiplication and division. Take the exercises in the order in which they are given. Work as many as you can and be