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**Five Place Logarithmic and Trigonometric Tables  
Arranged**

**Wentworth George Albert**

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**Title: Five Place Logarithmic and Trigonometric Tables Arranged**

**Author: Wentworth George Albert**

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PLANE AND SPHERICAL  
TRIGONOMETRY, SURVEYING  
AND TABLES

BY

G. A. WENTWORTH, A.M.

AUTHOR OF A SERIES OF TEXT-BOOKS IN MATHEMATICS

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REVISED EDITION

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## PREFACE.

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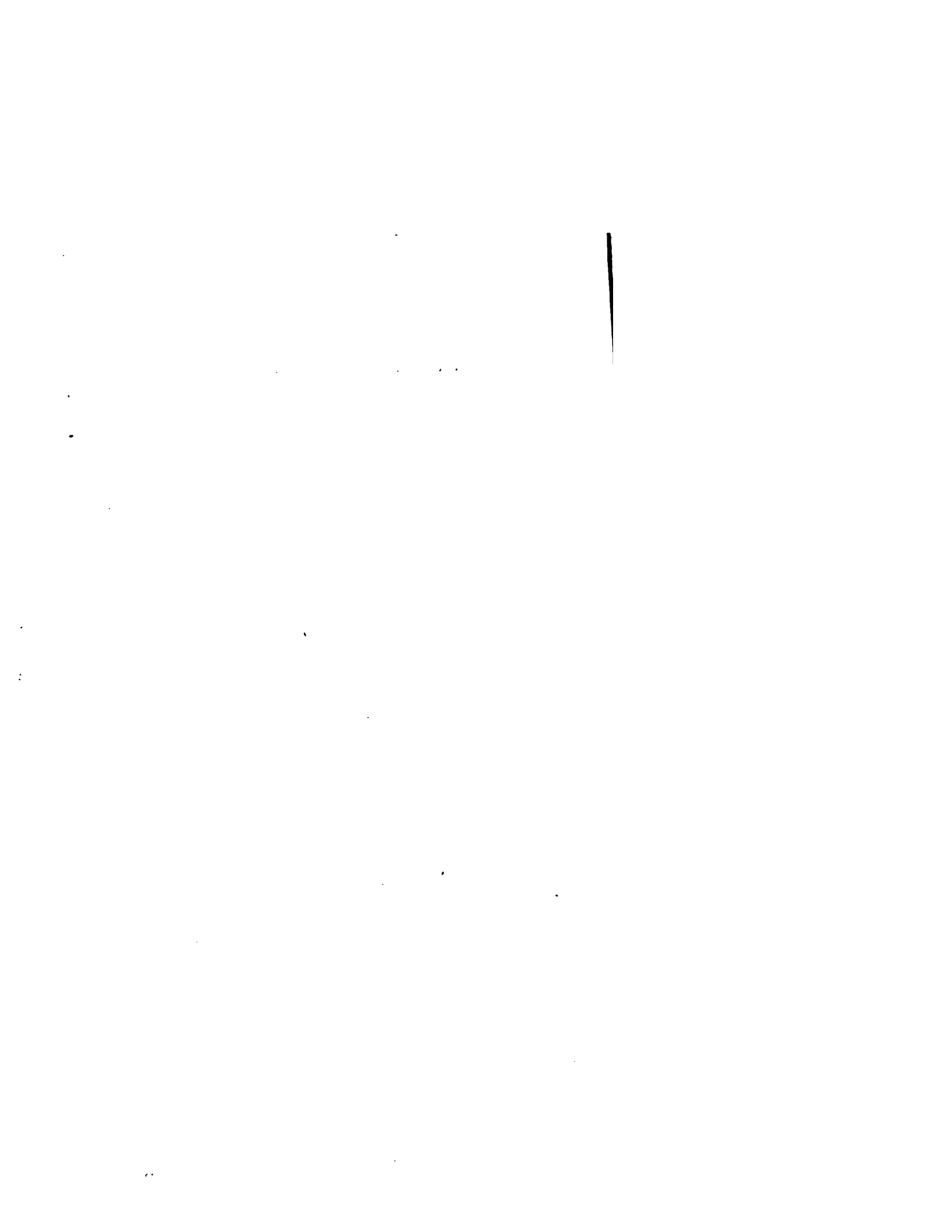
IN preparing this work the aim has been to furnish just so much of Trigonometry as is actually taught in our best schools and colleges. Consequently, all investigations that are important only for the special student have been omitted, except the development of functions in series. The principles have been unfolded with the utmost brevity consistent with simplicity and clearness, and interesting problems have been selected with a view to awaken a real love for the study. Much time and labor have been spent in devising the simplest proofs for the propositions, and in exhibiting the best methods of arranging the logarithmic work.

The object of the work on Surveying is to present this subject in a clear and intelligible way, according to the best methods in actual use; and also to present it in so small a compass that students in general may find the time to acquire a competent knowledge of this very interesting and important study.

The author is under particular obligation for assistance to G. A. Hill, A.M., of Cambridge, Mass., to Prof. James L. Patterson, of Schenectady, N.Y., to Dr. F. N. Cole, of Ann Arbor, Mich., and to Prof. S. F. Norris, of Baltimore, Md.

G. A. WENTWORTH.

EXETER, N.H., July, 1895



# CONTENTS.

## PLANE TRIGONOMETRY.

### CHAPTER I. FUNCTIONS OF ACUTE ANGLES :

Angular measure, page 1; trigonometric functions, 3; representation of functions by lines, 7; changes in the functions as the angle changes, 10; functions of complementary angles, 11; relations of the functions of an angle, 12; formulas for finding all the other functions of an angle, when one function of the angle is given, 15; functions of  $45^\circ$ ,  $30^\circ$ ,  $60^\circ$ , 17.

### CHAPTER II. THE RIGHT TRIANGLE :

Given parts of a triangle, 19. Solutions without logarithms, 19; Case I., when an acute angle and the hypotenuse are given, 19; Case II., when an acute angle and the opposite leg are given, 20; Case III., when an acute angle and an adjacent leg are given, 20; Case IV., when the hypotenuse and a leg are given, 21; Case V., when the two legs are given, 21. General method of solving a right triangle, 22; solutions by logarithms, 24; area of the right triangle, 26; the isosceles triangle, 31; the regular polygon, 33.

### CHAPTER III. GONIOMETRY :

Definition of goniometry, 36; angles of any magnitude, 36; general definitions of the functions of angles, 37; algebraic signs of the functions, 39; functions of a variable angle, 40; functions of angles greater than  $360^\circ$ , 42; formulas for acute angles extended to all angles, 43; reduction of the function of all angles to the functions of angles in the first quadrant, 46; functions of angles that differ by  $90^\circ$ , 48; functions of a negative angle, 49; functions of the sum of two angles, 51; functions of the difference of two angles, 53; functions of twice an angle, 55; functions of half an angle, 55; sums and differences of functions, 56.

### CHAPTER IV. THE OBLIQUE TRIANGLE :

Law of sines, 60; law of cosines, 62; law of tangents, 64. Solutions: Case I., when one side and two angles are given, 64; Case II.,



when two sides and the angle opposite to one of them are given, 66 ;  
Case III., when two sides and the included angle are given, 71 ; Case  
IV., when the three sides are given, 74 ; area of a triangle, 78-79.

**CHAPTER V. MISCELLANEOUS EXAMPLES :**

Plane Trigonometry, 82-99 ; goniometry, 99-105.

EXAMINATION PAPERS, 106-116.

**CHAPTER VI. CONSTRUCTION OF TABLES :**

Logarithms, 117 ; exponential and logarithmic series, 120 ; trigo-  
nometric functions of small angles, 125 ; Simpson's method of con-  
structing a trigonometric table, 127 ; De Moivre's theorem, 128 ;  
expansion of  $\sin x$ ,  $\cos x$ , and  $\tan x$ , in infinite series, 132.

---

**SPHERICAL TRIGONOMETRY.**

**CHAPTER VII. THE RIGHT SPHERICAL TRIANGLE :**

Introduction, 135 ; formulas relating to right spherical triangles,  
137 ; Napier's rules, 141. Solutions: Case I., when the two legs are  
given, 142 ; Case II., when the hypotenuse and a leg are given, 142 ;  
Case III., when a leg and the opposite angle are given, 143 ; Case IV.,  
when a leg and an adjacent angle are given, 143 ; Case V., when the  
hypotenuse and an oblique angle are given, 144 ; Case VI., when the  
two oblique angles are given, 144. The isosceles spherical triangle, 149.

**CHAPTER VIII. THE OBLIQUE SPHERICAL TRIANGLE :**

Fundamental formulas, 150 ; formulas for half angles and sides,  
152 ; Gauss's equations and Napier's analogies, 154. Solutions: Case  
I., when two sides and the included angle are given, 156 ; Case II.,  
when two angles and the included side are given, 158 ; Case III., when  
two sides and an angle opposite to one of them are given, 160 ; Case  
IV., when two angles and a side opposite to one of them are given,  
162 ; Case V., when the three sides are given, 163 ; Case VI., when  
the three angles are given, 164. Area of a spherical triangle, 166.

**CHAPTER IX. APPLICATIONS OF SPHERICAL TRIGONOMETRY :**

To reduce an angle measured in space to the horizon, 170 ; to find  
the distance between two places on the earth's surface, when the  
latitudes of the places and the difference in their longitudes are known,  
171 ; the celestial sphere, 171 ; spherical co-ordinates, 174 ; the astro-  
nomical triangle, 176 ; astronomical problems, 177-185.

**SURVEYING.****CHAPTER I. DEFINITIONS. INSTRUMENTS AND THEIR USES:**

Definitions, 135; instruments for measuring lines, 136; chaining, 136; obstacles to chaining, 138; the surveyor's compass, 141; uses of the compass, 143; verniers, 145; the surveyor's transit, 149; uses of the transit, 150; the theodolite, 150; the railroad compass, 150; plotting, 153.

**CHAPTER II. LAND SURVEYING:**

Determination of areas, 155; rectangular surveying, 159; field notes, computation, and plotting, 160; supplying omissions, 164; irregular boundaries, 164; obstructions, 164; modification of the rectangular method, 167; variation of the needle, 168; methods of establishing a true meridian, 170; dividing land, 173; United States public lands, 176; Burt's solar compass, 177; laying out the public lands, 179; Plane-table surveying, 181; the three-point problem, 186.

**CHAPTER III. TRIANGULATION:**

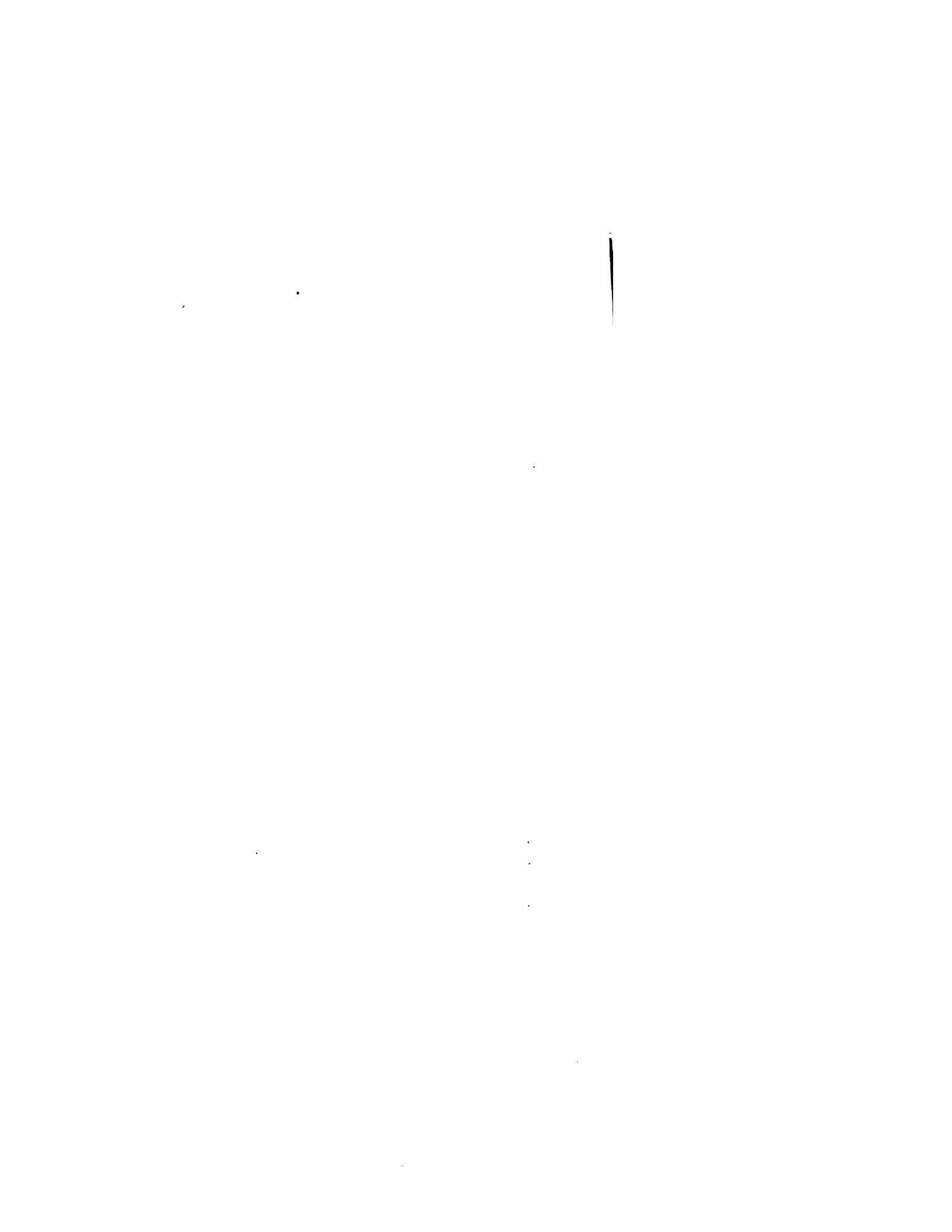
Introductory remarks, 187; the measurement of base lines, 188; the measurement of angles, 189.

**CHAPTER IV. LEVELLING:**

Definitions, 190; the Y level, 191; the levelling-rod, 191; difference of level, 192; levelling for section, 195; substitutes for the Y level, 198; topographical levelling, 200.

**CHAPTER V. RAILROAD SURVEYING:**

General remarks, 202; cross-section work, 202; railroad curves, 203.



# PLANE TRIGONOMETRY.

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## CHAPTER I.

### TRIGONOMETRIC FUNCTIONS OF ACUTE ANGLES.

#### § 1. ANGULAR MEASURE.

As lengths are measured in terms of various conventional units, as the foot, meter, etc., so different units for measuring angles are employed, or have been proposed.

In the common or *sexagesimal* system the circumference of a circle is divided into 360 equal parts. The angle at the centre subtended by each of these parts is taken as the unit angle and is called a *degree*. The degree is subdivided into 60 *minutes*, and the minute into 60 *seconds*. A right angle is equal to 90 degrees.

**NOTE.** The *sexagesimal* system was invented by the early Babylonian astronomers in conformity with their year of 360 days.

In the *circular* system an arc of a circle is laid off equal in length to the radius. The angle at the centre subtended by this arc is taken as the unit angle and is called a *radian*.

The number of radians in  $360^\circ$  is equal to the number of times the length of the radius is contained in the circumference. It is proved in Geometry that this number is  $2\pi$  ( $\pi = 3.1416$ ) for all circles; therefore the radian is the same angle in all circles.