Plane and Spherical Trigonometry

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PLANE AND SPHERICAL

TRIGONOMETRY.

BY

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BOSTON, U.S.A.:
PUBLISHED BY GINN & COMPANY.
1889.
Enter., according to Act of Congress, in the year 1883, by
G. A. WENTWORTH,

TYPOGRAPHY BY J. S. CUSHING & CO., BOSTON, U.S.A.
PRESSWORK BY GINN & CO., BOSTON, U.S.A.
PREFACE.

In preparing this work the aim has been to furnish, just so much of Trigonometry as is actually taught in our best schools and colleges. Consequently, development of functions in series and all other investigations that are important only for the special student have been omitted. The principles have been unfolded with the utmost brevity consistent with simplicity and clearness, and interesting problems have been selected with a view to awaken a real love for the study. Much time and labor have been spent in devising the simplest proofs for the propositions, and in exhibiting the best methods of arranging the logarithmic work.

The object of the work on Surveying and Navigation is to present these subjects in a clear and intelligible way, according to the best methods in actual use; and also to present them in so small a compass that students in general may find the time to acquire a competent knowledge of these very interesting and important studies.

The author is under particular obligation for assistance to G. A. Hill, A.M., of Cambridge, Mass., and to Prof. James L. Patterson, of Lawrenceville, N.J.

G. A. WENTWORTH.

PHILLIPS EXETER ACADEMY,
September, 1882.
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PLANE TRIGONOMETRY.

CHAPTER I.

TRIGONOMETRIC FUNCTIONS OF ACUTE ANGLES.

§ 1. DEFINITIONS.

The sides and angles of a plane triangle are so related that any three given parts, provided at least one of them is a side, determine the shape and the size of the triangle.

Geometry shows how, from three such parts, to construct the triangle and find the values of the unknown parts.

Trigonometry shows how to compute the unknown parts of a triangle from the numerical values of the given parts.

Geometry shows in a general way that the sides and angles of a triangle are mutually dependent. Trigonometry begins by showing the exact nature of this dependence in the right triangle, and for this purpose employs the ratios of its sides.

Let MAN (Fig. 1) be an acute angle. If from any points $B, D, F, \ldots$ in one of its sides perpendiculars $BC, DE, FG, \ldots$ are let fall to the other side, then the right triangles $ABC, ADE, AFG, \ldots$ thus formed have the angle $A$ common, and are therefore mutually equiangular and similar. Hence, the ratios of their corresponding sides, pair by pair, are equal. That is,

\[
\frac{AC}{AB} = \frac{AE}{AD} = \frac{AG}{AE} \quad \frac{AC}{BC} = \frac{AE}{DE} = \frac{AG}{FG}; \quad \text{etc.}
\]