
Plane Trigonometry and Tables

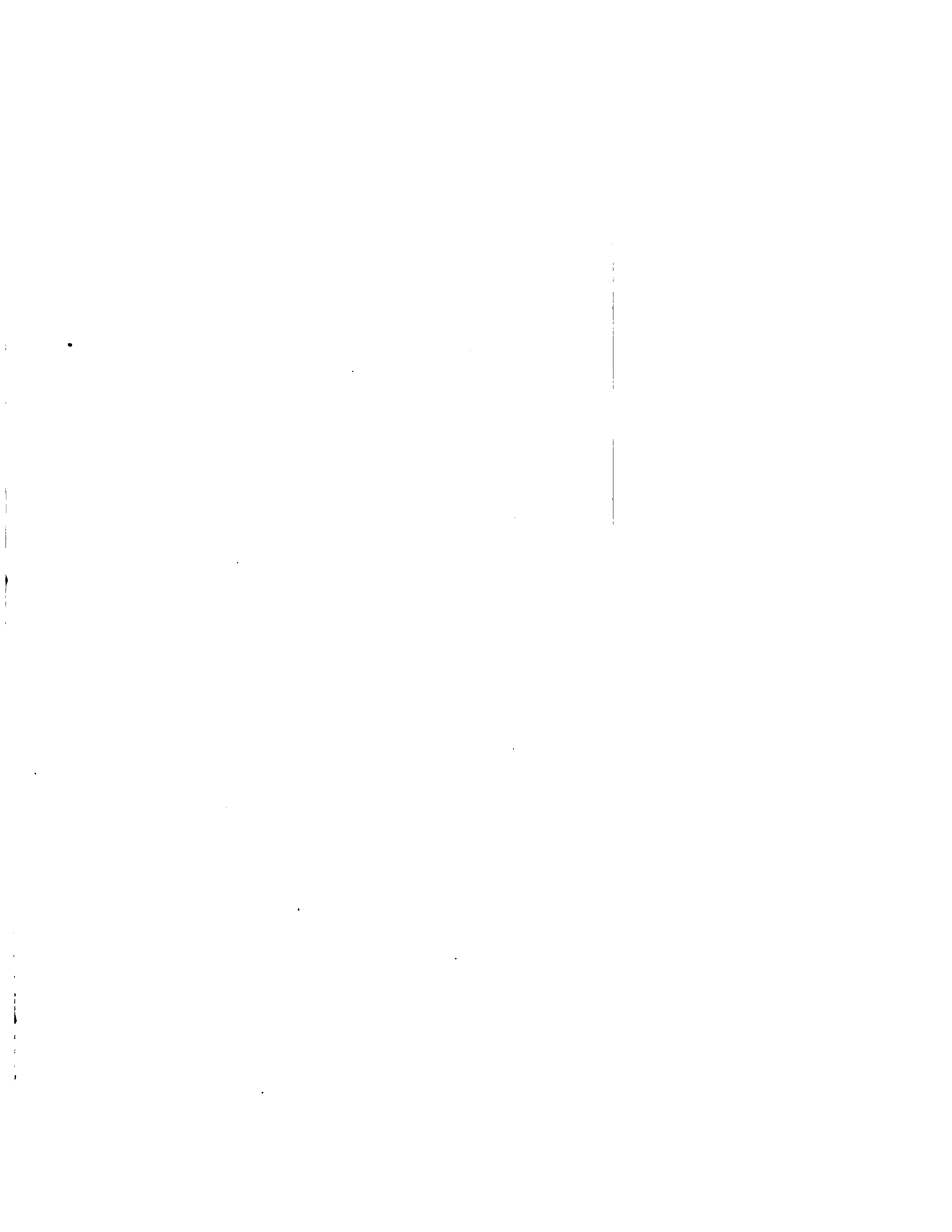
Wentworth George Albert

Title: Plane Trigonometry and Tables

Author: Wentworth George Albert

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PLANE
TRIGONOMETRY, SURVEYING
AND
TABLES

BY
G. A. WENTWORTH
AUTHOR OF A SERIES OF TEXT-BOOKS IN MATHEMATICS

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PREFACE.

IN preparing this work the aim has been to furnish just so much of Trigonometry as is actually taught in our best schools and colleges. Consequently, all investigations that are important only for the special student have been omitted, except the development of functions in series. The principles have been unfolded with the utmost brevity consistent with simplicity and clearness, and interesting problems have been selected with a view to awaken a real love for the study. Much time and labor have been spent in devising the simplest proofs for the propositions, and in exhibiting the best methods of arranging the logarithmic work.

The author is under particular obligation for assistance to G. A. Hill, A.M., of Cambridge, Mass., to Prof. James L. Patterson, of Schenectady, N.Y., to Dr. F. N. Cole, of Ann Arbor, Mich., and to Prof. S. F. Norris, of Baltimore, Md.

G. A. WENTWORTH.

EXETER, N.H., July, 1895.

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PLANE TRIGONOMETRY.

CHAPTER I.

TRIGONOMETRIC FUNCTIONS OF ACUTE ANGLES.

§ 1. ANGULAR MEASURE.

As lengths are measured in terms of various conventional units, as the foot, meter, etc., so different units for measuring angles are employed, or have been proposed.

In the common or *sexagesimal* system the circumference of a circle is divided into 360 equal parts. The angle at the centre subtended by each of these parts is taken as the unit angle and is called a *degree*. The degree is subdivided into 60 *minutes*, and the minute into 60 *seconds*. A right angle is equal to 90 degrees.

NOTE. The sexagesimal system was invented by the early Babylonian astronomers in conformity with their year of 360 days.

In the *circular* system an arc of a circle is laid off equal in length to the radius. The angle at the centre subtended by this arc is taken as the unit angle and is called a *radian*.

The number of radians in 360° is equal to the number of times the length of the radius is contained in the circumference. It is proved in Geometry that this number is 2π ($\pi = 3.1416$) for all circles; therefore the radian is the same angle in all circles.

Since the circumference of a circle is 2π times the radius,

$$2\pi \text{ radians} = 360^\circ, \text{ and } \pi \text{ radians} = 180^\circ;$$

Therefore, $1 \text{ radian} = \frac{180^\circ}{\pi} = \frac{180^\circ}{3.1416} = 57^\circ 17' 45''$

and $1 \text{ degree} = \frac{\pi}{180} \text{ radian} = 0.017453 \text{ radian}.$

By the last two equations the measure of an angle can be changed from radians to degrees or from degrees to radians.

Thus, $2 \text{ radians} = 2 \times \frac{180^\circ}{\pi} = 2 \times (57^\circ 17' 45'') = 114^\circ 35' 30''.$

NOTE. The circular system came into use early in the last century. It is found more convenient in the higher mathematics, where the radians are simply expressed as numbers. Thus the angle π means π radians, and the angle 3 means 3 radians.

On the introduction of the metric system of weights and measures at the close of the last century, it was proposed to divide the right angle into 100 equal parts called *grades*, which were to be taken as units. The grade was subdivided into 100 *minutes* and the minute into 100 *seconds*. This *French* or *centesimal* system, however, never came into actual use.

EXERCISE I.

[Assume $\pi = 3.1416$.]

1. Reduce the following angles to circular measure, expressing the results as fractions of π . 60° , 45° , 150° , 195° , $11^\circ 15'$, $123^\circ 45'$, $37^\circ 30'$.

2. How many degrees are there in $\frac{2}{3}\pi$ radians? $\frac{3}{4}\pi$ radians?

$\frac{5}{8}\pi$ radians? $\frac{15}{16}\pi$ radians? $\frac{7}{15}\pi$ radians?

3. What decimal part of a radian is 1° ? $1'$?

4. How many seconds in a radian?