Key to Plane Geometry

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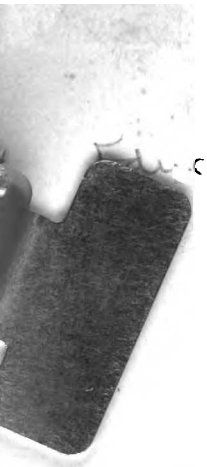

KEY

TO

PLANE GEOMETRY

BY

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KEY TO PLANE GEOMETRY

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17. He first sets out two of the trees in position and then sights through them and places the other trees in line with the two already set out.

22. Straight, *I*; curved, *C*, *O*, *Q*, *S*, *U*; broken, *A*, *E*, *F*, *H*, *K*, *L*, *M*, *N*, *T*, *V*, *W*, *X*, *Y*, *Z*; mixed, *B*, *D*, *G*, *J*, *P*, *R*.

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3. 47°; 137°.

4. 32° 41′; 27° 36′ 17″; 122° 41′; 117° 36′ 17″.

5. Construct a right angle having the same vertex as the given angle and a side in common with this angle, and including the given angle as a part of itself.
To construct the supplement, produce a side of the given angle through the vertex.

6. Obtuse.

7. 6°.

8. 60°; 90°; 150°.

9. 135°; 22° 30′; 7° 30′.

10. 10 min.; 8⅓ min.; 40 min.; 2 hr.; 1 hr. 40 min.; 8 hr.

11. 810°.

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12. $\frac{5}{6}$.

13. 30°; 90°; 22½°; 15°; 1¼°.

14. 1 lb.; ⅔ lb.; 1½ lb.; 2⅔ lb.; $\frac{1}{10}$ lb.; 6 lb.

15. Acute; obtuse; right.

16. Obtuse; acute.

19. Reflex; acute.

20. Right; obtuse; acute.

21. Right angle, for, denote the given angle by x. Then $(180°-x) - (90° - x) = 90°$.

22. (*a*) $r = t$. (*b*) $r = t$. (*c*) $t > r$, for s and r are acute. $\therefore t$ is obtuse. (*d*) $r > t$, for s is acute and $\therefore r$ is obtuse.

23. (*a*) $m > p$. (*b*) $m = p$. (*c*) $m < p$. (*d*) Denote the comp. of m by x. Then $x + m = 90°$, $x + p = 180°$. $\therefore p - m = 90°$. $\therefore p = m + 90°$. $\therefore p > m$.

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24. $x = 2(90° - x)$. $\therefore\ x = 60°$. *Ans.*

25. $x = \frac{1}{3}(180° - x)$. $\therefore\ x = 45°$. *Ans.*

26. $x = \frac{4}{5}(180° - x)$. $\therefore\ x = 80°$. *Ans.*

27. (1) $x = 90° - x + 12°$. $\therefore\ x = 51°$. *Ans.*
(2) $x = 180° - x + 15\frac{1}{2}°$. $\therefore\ x = 97°\,45'$. *Ans.*

28. $90° - x + 180° - x = 126°$. $\therefore\ x = 72°$. *Ans.*

29. $180° - x = 4(90° - x)$. $\therefore\ x = 60°$. *Ans.*

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2. Yes, by placing the edge of the ruler on the pipe in a direction parallel to the length of the pipe. No.

3. No.

4. Place the straight edge on the surface in various directions. In all positions every point of the straight edge should be in contact with the surface of the tennis court.

5. Yes.

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1. Ax. 7.

2. 140°; 210°; Ax. 7.

3.
$$\begin{array}{r} 7 = 7 \\ -2 = -2 \\ \hline 5 = 5 \end{array}$$

4. (1)
$$\begin{array}{r} 5 = 5 \\ \times 2 \quad \times 2 \\ \hline 10 = 10 \end{array}$$
(2) $12 = 12$. Dividing each of these by 3, $4 = 4$.

5. $8 = 8$. $\therefore\ \sqrt[3]{8} = \sqrt[3]{8}$, or $2 = 2$.

6. Ax. 4.

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7. Ax. 2.

8. Use the diagram of Ex. 7. Thus, if $LN = MO$, then $LM = NO$ (Ax. 3).

11. Ax. 1. **12.** Ax. 2. **13.** Ax. 9.

14. Through two points only one straight line can be passed; and a right angle is half of a straight angle.

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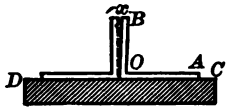
1. Three.
2. Six.
3. In Ex. 1 each point helps to locate two lines; in Ex. 2, each point helps to locate three lines. Hence, a point in the latter case does $1\frac{1}{2}$ times as much work, or is $1\frac{1}{2}$ times as efficient as in the former case.
5. 140.° 6. $r = 43°$, etc.
8. $\angle ABP = 40°$; $\angle PBQ = 50°$; $\angle QBC = 40°$.
9. $\angle COD = 180° - 142° = 38°$. $\angle COB = 120° - 38° = 82°$. *Ans.*
10. 

Place AO, one of the inside edges of the square, in contact with the straight edge CD. Then repeat the act, placing O at the same point on CD, with OA pointed in the opposite direction from the first position.

By § 23, the sum of two right angles must be a straight angle.

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2. Any two of the angles which are not vertical.
3. 157° (§ 69).
4. $s = p = 153° - 107° = 46°$. *Ans.*
5. $t = q = 138°$ (see Ex. 9, p. 28).
6. $4r = 180° - 24°$. $\therefore r = x = 39°$. *Ans.*

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2. 72°; 104°.
3. Add the complement to 90°; subtract 90° from the supplement.
4. 150°; 12° 30′.
5. As the sides of a triangle; two || lines, one through each point, and a third line through the two given points; etc.
6. 12 in. 7. No. 8. No.
10. Denote the angle of the square by y. Then $2(y + e) = 2y + x$. $\therefore 2e = x$. $\therefore e = \frac{1}{2}x$.

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1. $EF = 18$; $\angle E = 70°$; $\angle F = 50°$.

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1. $DE = 24$; $EF = 27$; $\angle E = 55°$.

1. (Group 9). Use § 79.

2. § 79. 3. § 79. 4. § 79.

5. Prove $\triangle DCF = \triangle ACB$ by § 79. 217 yd.

6. § 80.

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7. § 80. 8. § 80.

9. $\angle ABO = \angle CBO$ by § 66. Use § 80.

10. Use § 80. 137 yd.

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1. By the principle proved in § 82, $\angle C = \angle A = 67°$.

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1. $\angle BAC = \angle BCA$ (§ 82). $\therefore p = r$ (§ 66).
Then prove $\triangle DAB = \triangle BCF$ by § 79.

2. Use § 82 twice and Ax. 3.

3. § 83. 4. § 83.

5. Draw AC and use § 83.

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2. Construct a right angle and bisect it. Also through the vertex draw a line $\perp$ the bisector.

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2. No, for $d = 110°$, and hence d and f are not supplementary. Use § 99.

1. (Group 11). $\angle MOQ = \angle POL$ (§ 69). $PO = OQ$ (Hyp.).
$\angle MQO = \angle OPL$ (§ 96). $\therefore \triangle POL = \triangle OMQ$ (§ 80).

2. $\angle A = \angle C$ (§ 82); $\angle D = \angle C$ (§ 96), etc.

3. Prove $\triangle ABP = \triangle PCD$ by § 80, etc.

4. $x = p$ (§ 69); $y = q$ (§ 69); $x = y$ (Ax. 1). $\therefore AB \parallel CD$ (§ 89).

5. $\angle ABC = \angle BCD$ (Ax. 2), etc.

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6. $y = m$ (§ 82). $x = y$ (§ 97); $l = m$ (§ 97), etc.
7. Prove $\triangle ABD = \triangle BFC$ by § 79, etc.
8. $\angle B = \angle C$ (§ 82). Then use § 96 twice and Ax. 1.
9. $\angle CBE = 115°$ (Ax. 7). $\therefore$ $\angle ABE = 65°$ (§ 32).
   $\therefore$ $BE \parallel CD$ (§ 91).
10. $\angle B = \angle i$ (step 6, p. 43), etc.

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1. 62°.
2. 53° 45′.
3. 60°.
4. 45°.
5. No.

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6. 138°.
7. 142°; 115°; 103°.
8. 71°.
9. 80°.
10. 78°; 78°; 24°.
11. Construct an equilateral triangle and bisect one of its angles. Bisect an angle of 30°.
12. Construct the supplement of 60°. $75° = 45° + 30°$.
13. $150° = 90° + 60°$. $195° = 180° + 15°$.
15. Construct an equilateral triangle and a perpendicular to the base through an extremity of the base.
16. See Ex. 12.
17. Construct an angle of 45° at each end of the 2-in. line.
19. Corr. $\measuredangle$ are $=$ (§ 107). $\blacktriangle$ are not equal.
20. Through the vertex of the acute angle construct a $\perp$ to one side of the angle.
21. Produce one side of the angle through the vertex.

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1. Use § 114.
2. § 114.
3. $\angle B$ (§ 114).
4. § 114.
5. $\angle OAQ = \angle OBP$ (§ 114).
   $\angle AOQ = \angle BOP$ (§ 69).
   $\angle AOB = \angle POQ$ (§ 69).
   $\angle AQO = \angle OQC = \angle APC = \angle APB$ (§ 63).
6. See Ex. 1, p. 39.