Some graphical solutions of electric railway problems

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SOME GRAPHICAL SOLUTIONS OF ELECTRIC RAILWAY PROBLEMS

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SOME GRAPHICAL SOLUTIONS OF ELECTRIC RAILWAY PROBLEMS

I. INTRODUCTION

In the solution of railway problems involving the characteristics of the motive power it is difficult to use analytical methods, principally because it is impossible to obtain a satisfactory general equation for the curves of an engine or motor of any specified type. The relation between speed and tractive effort, for instance, is so involved that any attempt to obtain a formula leads to assumptions which cannot be made without seriously affecting the accuracy of the final result.* This is true not only of the steam locomotive, but also of the various types of electric motors ordinarily used for train propulsion.

The graphical methods, in contrast with the analytical, form an accurate and at the same time an easy means of attack applicable to any possible combination of characteristics and any range of conditions which may be met in practice. It is the purpose of this bulletin to develop a number of new graphical methods which, in connection with other well-known ones, aid materially in the solution of such problems. While most of these were developed in connection with problems of electric train performance, a number of them are equally applicable to any type of motive power, a fact which is set forth in the paragraphs which follow.

The majority of these methods were developed by the writer in connection with classroom instruction. One of the ways of obtaining motor performance with varying potential and one for finding the "effective" value of the motor current are due to Mr. S. Sekine, a graduate student in Railway Engineering in the University of Illinois, who is also responsible for a portion of the method of plotting speed-time and distance-time curves.

II. MOTOR PERFORMANCE WITH VARYING POTENTIAL†

The performance characteristics of a railway motor are ordinarily furnished by the manufacturer for the normal potential and are usually assumed to be accurate under such conditions. Often it is desirable to find the motor performance when abnormal potential is impressed on the terminals, since in practice the line pressure is subject to wide fluctuations, and the motors are always operating at subnormal potential while the controller is being turned to the full-speed position.

†For a brief discussion of this topic see Electric Railway Journal, Sept. 18, 1915.
The torque produced by a given current in a series motor is practically independent of the line pressure, so that recalculation of this quantity is unnecessary for any ordinary conditions of operation met with in practice, unless, of course, the field strength is purposely reduced. The only other important variable to be considered is the motor speed.

In an electric motor the applied pressure is used up in two ways; a portion overcomes the drop due to the resistance of the windings, and the remainder opposes the counter e.m.f. generated in the armature. If the field flux remains constant, the speed will vary in direct proportion to the counter e.m.f. which is developed. This may be expressed by the equation

\[
\frac{V_2}{V_1} = \frac{E_2 - Ir}{E_1 - Ir}
\]  
\( \cdots \cdots \cdots \cdots \cdots \cdots (1) \)

in which \( V_1 \) and \( V_2 \) are the speeds when \( E_1 \) volts and \( E_2 \) volts are applied at the terminals, respectively, \( I \) is the current flowing through the armature, and \( r \) is the motor resistance, or that portion in the armature and the circuits in series therewith.

**Fig. 1. Volt-Ampere Diagram for Electric Motor.**

In order to make the calculation graphically it is only necessary to determine the relative values of \( E_1 - Ir \) and \( E_2 - Ir \), from which the ratio of speeds may be found directly. A simple method of showing the relations between these values is to construct a diagram with motor volts as ordinates and armature amperes as abscissae, as

*A. M. Buck, The Electric Railway, p. 53.*
shown in Fig. 1. Since the \( Ir \) drop is a direct function of the armature current, it can be represented for all values of current by the intercepts on a straight line with the proper slope. This may be drawn through the origin, but, since we are principally concerned with the difference between the terminal pressure and the \( Ir \) drop, it is better to draw it from the line of full pressure at the motor terminals, \( E_1 \). If the terminal pressure is then changed to \( E_2 \) volts, it will not affect the slope of the \( Ir \) line, but will change its position so that it begins at the point \( E_2 \). In each case the counter e.m.f. is the residue after subtracting the \( Ir \) drop, as shown in the diagram. All that remains is to obtain a graphical relation between \( V_1 \) and \( V_2 \), which is proportional to these values of counter e.m.f. Two methods of doing this have been developed.

The first method of calculation is shown in Fig. 2. Here the volt-ampere diagram of Fig. 1 is reproduced, along with the speed-current curve of the motor; as determined by test or from design calculations, the axes of current being in the same straight line. The current scales and their positions along the axis may be chosen as desired, their relation to each other being immaterial. The speed of the motor at the terminal pressure \( E_1 \) is represented by the ordinate \( V_1 \). It is desired to find the corresponding value of speed \( V_2 \) at \( E_2 \) volts and the same current \( I \). Draw a line through \( A \) at the value of current \( I \) on the volt-ampere diagram and also through \( V_1 \). This
will intersect the axis of abscissae at some point $K$. From $K$ draw the line $KB$, through the corresponding point $B$ on the volt-ampere diagram for the same current and the new pressure $E_2$. This locates $V_2$, the speed at $E_2$ volts, at the intersection of $KB$ with the current ordinate. It must be correct since, by similar triangles,

\[
\frac{IV_1}{IV_2} = \frac{IA}{IB}
\]

It may be seen from Fig. 1 that $IA$ and $IB$ are the values of counter e.m.f. corresponding to the pressures $E_1$ and $E_2$ at the current $I$

It should be noted that a different position of the point $K$ will be located for each value of current, and in some cases it may be at too great a distance from the body of the diagram. To obviate this the relative positions of the speed-current and the volt-ampere diagrams may be changed, always keeping their current axes together.

In some cases it is preferable to make the entire construction on the speed current diagram. The arrangement for this method is shown in Fig. 3. Here the base of the volt-ampere diagram is taken the same as that for the speed-current curve, and the propor-

![Figure 3](image)

**Fig. 3. Second Method for Obtaining Motor Speeds at Different Potentials.**

The two projections of the values of counter e.m.f. will meet at some point, such as $P$, and a line drawn connecting $P$ with the origin will
divide the ordinate and abscissa of any point along it proportionally to these two values. Then, by projecting the speed at $E_1$ volts onto this line, the speed at $E_2$ volts and the same current are given by the corresponding abscissa, and may be carried back through 90 degrees and plotted on the original current ordinate, as shown.

A further inspection of Fig. 3 shows that the locus of the point $P$ will be a line $MN$, which passes through $N$, corresponding to zero $Ir$ drop, and makes an angle of 45 degrees with the axes. The proof of this construction is that the $Ir$ drop is the same for a given current irrespective of the terminal pressure. For this reason it is unnecessary to swing mechanically the counter e.m.f. line through 90 degrees to locate $P$. Draw $MN$ from the intersection $N$ of the projections of $E_1$ and $E_2$ (taken at right angles, as explained above). Any point on the counter e.m.f. line will then give a projection on $MN$, as at $P$, thus saving the preliminary construction.

III. Motor Performance with Resistance

To determine the performance of a motor when a resistance is inserted in series with the armature, the constructions given in Figs. 2 and 3 may be used with a slight modification. Fig. 4 is the

![Graphical representation of motor performance with resistance.](image)

**Fig. 4. First Method for Obtaining Motor Speeds with Resistance.**

same as Fig. 2, except that the $Ir$ drop at a different pressure has been replaced by a line $E_1B$ representing the drop $I(R + r)$, in which $R$ is the external resistance in the circuit. The procedure is the
same as that explained in the determination of motor performance with varying potential, and the proof of the construction is identical. The method of Fig. 3 can equally well be used for determining motor speeds with resistance, as shown in Fig. 5. Since the $IR$ drop

![Fig. 5. Second Method for Determining Motor Speeds with Resistance.](image)

is not the same, the line $MN$ has a different angle, which is determined by the relative values of resistance in the two cases; that is, if the line $MN$ of Fig. 5 makes an angle $\theta$ with the axis of abscissae,

$$\tan \theta = \frac{r}{R + r}$$

(3)

With this modification the method is precisely the same as that described above.

IV. Starting Resistance for Series Motors with Rheostatic Control

In starting direct-current series motors it is usually not sufficient to reduce the potential at the motor terminals by making different combinations of motors on the supply circuit. When this can be done, as may be possible with very small motors, the performance may be predicted by calculating the performance curves at the lower potentials, as described previously in this bulletin, or by any other ordinary method. In general, however, it is necessary to place a certain external resistance in the circuit, whether or not the potential