Treatise on natural philosophy
Kelvin William Thomson
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Author: Kelvin William Thomson

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TREATISE
ON
NATURAL PHILOSOPHY

BY
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AND
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PART II.

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PREFACE.

The original design of the Authors incommencing this work about twenty years ago has not been carried out beyond the production of the first of a series of volumes, in which it was intended that the various branches of mathematical and experimental physics should be successively treated. The intention of proceeding with the other volumes is now definitely abandoned; but much new matter has been added to the first volume, and it has been divided into two parts, in the second edition now completed in this second part. The original first volume contained many references to the intended future volumes; and these references have been allowed to remain in the present completion of the new edition of the first volume, because the plan of treatment followed depended on the expectation of carrying out the original design.

Throughout the latter part of the book extensive use has, according to Prof. Stokes' revival of this valuable notation, been made of the "solidus" to replace the horizontal stroke in fractions; for example $\frac{a}{b}$ is printed $a/b$. This notation is (as is illustrated by the spacing between these lines) advantageous for the introduction of isolated analytical expressions in the midst of the text, and its use in printing complex fractional and exponential expressions permits the printer to dispense with much of the troublesome process known as "justification," and effects a considerable saving in space and expense.
An index to the whole of the first volume has been prepared by Mr Burnside, and is placed at the end.

A schedule is also given below of all the amendments and additions (excepting purely verbal changes and corrections) made in the present edition of the first volume.

Inspection of the schedules on pages xxii. to xxv. will shew that much new matter has been imported into the present edition, both in Part I. and Part II. These additions are indicated by the word “new.”

The most important part of the labour of editing Part II. has been borne by Mr G. H. Darwin, and it will be seen from the schedule below that he has made valuable contributions to the work.

NOTE TO NEW IMPRESSION, 1912

A few slight additions and corrections have been made by Sir George Darwin and Prof. H. Lamb, but, substantially, the work remains as last passed by the authors. The additions can be identified by the initials attached in brackets.

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