A treatise on electricity and magnetism

Maxwell James Clerk
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Author: Maxwell James Clerk

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A TREATISE

ON

ELECTRICITY AND MAGNETISM

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A TREATISE
ON
ELECTRICITY AND MAGNETISM

BY

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