Elementary theory of equations

Dickson Leonard E
Title: Elementary theory of equations

Author: Dickson Leonard E

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College Algebra.
A textbook for colleges and technical schools.
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Algebraic Invariants.

BY
G. A. MILLER, H. F. BLICHFELDT
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By G. A. Miller, Professor of Mathematics in the University of Illinois; H. F. Blichfeldt, Ph. D., Professor of Mathematics in Stanford University; and L. E. Dickson. 390 pages, 6 x 9. Illustrated. Cloth.

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73 pages, Cambridge University Press.

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History of the Theory of Numbers.
ELEMENTARY THEORY
OF
EQUATIONS

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PREFACE

The longer an engineer has been separated from his alma mater, the fewer mathematical formulas he uses and the more he relies upon tables and, when the latter fail, upon graphical methods. Although graphical methods have the advantage of being ocular, they frequently suffer from the fact that only what is seen is sensed. But this defect is due to the kind of graphics used. With the aid of the scientific art of graphing presented in Chapter I, one may not merely make better graphs in less time but actually draw correct negative conclusions from a graph so made, and therefore sense more than one sees. For instance, one may be sure that a given cubic equation has only the one real root seen in the graph, if the bend points lie on the same side of the x-axis.

Emphasis is here placed upon Newton’s method of solving numerical equations, both from the graphical and the numerical standpoint. One of several advantages (well recognized in Europe) of Newton’s method over Horner’s is that it applies as well to non-algebraic as to algebraic equations.

In this elementary book, the author has of course omitted the difficult Galois theory of algebraic equations (certain texts on which are very erroneous) and has merely illustrated the subject of invariants by a few examples.

It is surprising that the theorems of Descartes, Budan, and Sturm, on the real roots of an equation, are often stated inaccurately. Nor are the texts in English on this subject more fortunate on the score of correct proofs; for these reasons, care has been taken in selecting the books to which the reader is referred in the present text.

The material is here so arranged that, before an important general theorem is stated, the reader has had concrete illustrations and often also special cases. The exercises are so placed that a reasonably elegant and brief solution may be expected, without resort to tedious multiplications and similar manual labor. Very few of the five hundred exercises are of the same nature.

Complex numbers are introduced in a logical and satisfying manner. The treatment of roots of unity is concrete, in contrast to the usual abstract method.

Attention is paid to scientific computation, both as to control of the limit of error and as to securing maximum accuracy with minimum labor.

An easy introduction to determinants and their application to the solution of systems of linear equations is afforded by Chapter XI, which is independent of the earlier chapters.
Here and there are given brief, but clear, outlooks upon various topics of decided intrinsic and historical interest, — thus putting real meat upon the dry bones of the subject.

To provide for a very brief course, certain sections, aggregating over fifty pages, are marked by a dagger for omission. However, in compensation for the somewhat more advanced character of these sections, they are treated in greater detail.

In addition to the large number of illustrative problems solved in the text, there are five hundred very carefully selected and graded exercises, distributed into seventy sets. As only sixty of these exercises (falling into seventeen sets) are marked with a dagger, there remains an ample number of exercises for the briefer course.

The author is greatly indebted to his colleagues Professors A. C. Lunn and E. J. Wilczynski for most valuable suggestions made after reading the initial manuscript of the book. Useful advice was given by Professor G. A. Miller, who read part of the galley proofs. A most thorough reading of both the galley and page proofs was very generously made by Dr. A. J. Kempner, whose scientific comments and very practical suggestions have led to a marked improvement of the book. Moreover, the galleys were read critically by Professor D. R. Curtiss, who gave the author the benefit not merely of his wide knowledge of the subject but also of his keen critical ability. The author sends forth the book thus emended with less fear of future critics, and with the hope that it will prove as stimulating and useful as these five friends have been generous of their aid.

Chicago, February, 1914.
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